

# Electrical Power Engineering (2)

Code: EP2207

Lecture: 4

Tutorial: 4

Total: 8

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# Network Equations and Solution

The complexity of power systems makes ordinary solution of the necessary network equations impractical

The digital computers are used to do calculations for all problems

The solution in digital computers depends on the network equations

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# Network Equations and Solution

The number of equations describing large interconnected networks is extremely high

This large number of equations can be reduced by nodes elimination

Caution has to be taken to eliminate nodes that are not important and do not affect the solution

Elimination of a node is avoided when a knowledge of the voltage is a desirable part of the solution

In cases where the voltage at a particular node is unimportant, the node can be eliminated

Voltage at the eliminated node and currents related to the node can be found by additional calculations

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# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations

If many stability studies are to be made on a system, node elimination reduces the number of calculations and thus it is advantageous since such studies seldom call for information about bus voltages

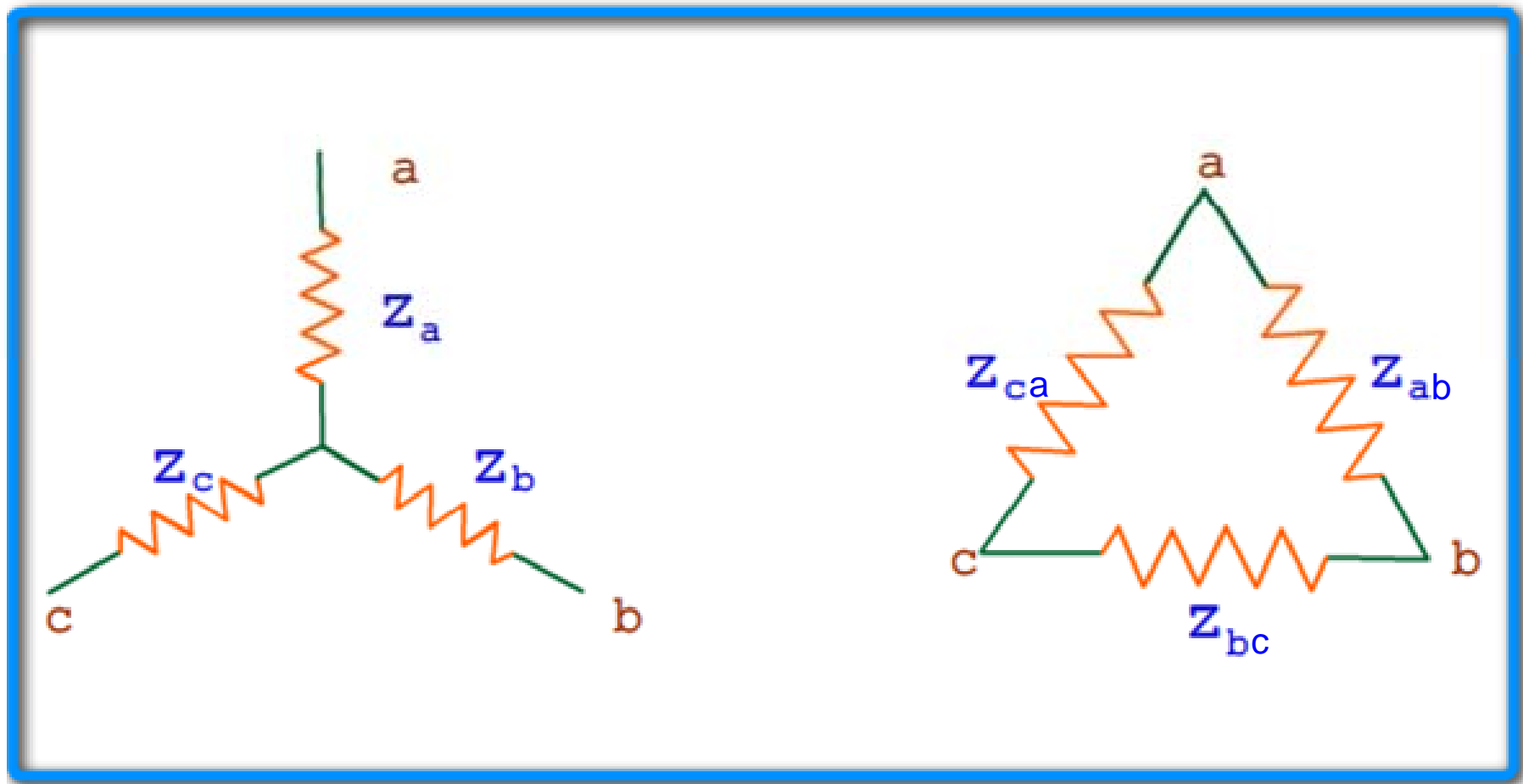
If only three elements terminate at a node without any sources, the node is eliminated by a Y- $\Delta$  transformation

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# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations

### Y to $\Delta$ transformation



# Network Equations and Solution

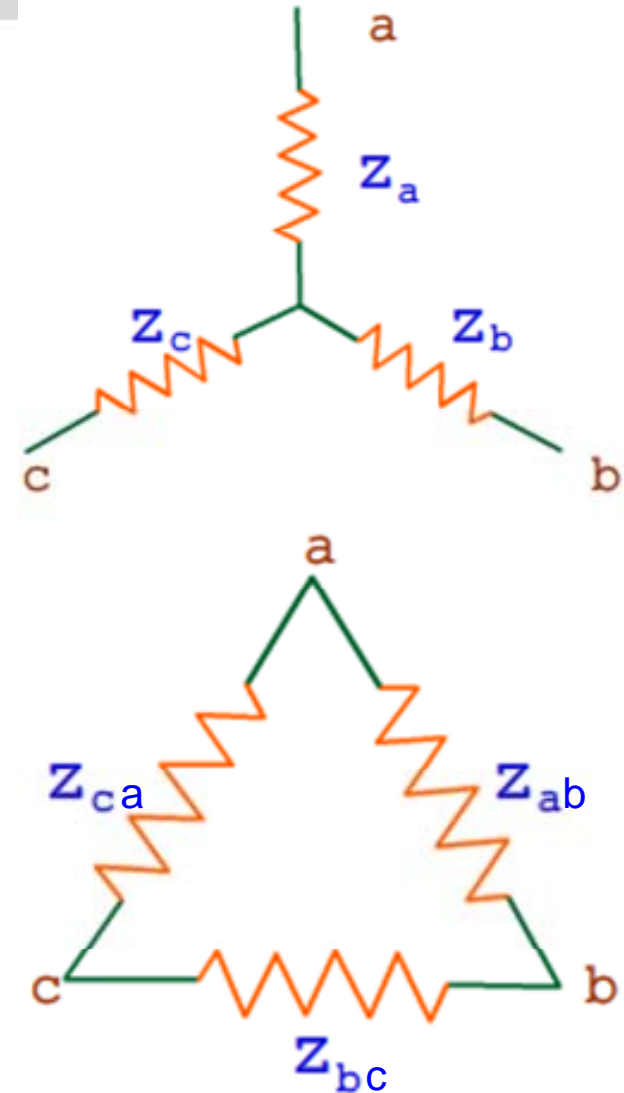
## Node Elimination by Star-Mesh Transformations

Y to  $\Delta$  transformation

$$Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}$$

$$Z_{bc} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a}$$

$$Z_{ca} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b}$$



# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations

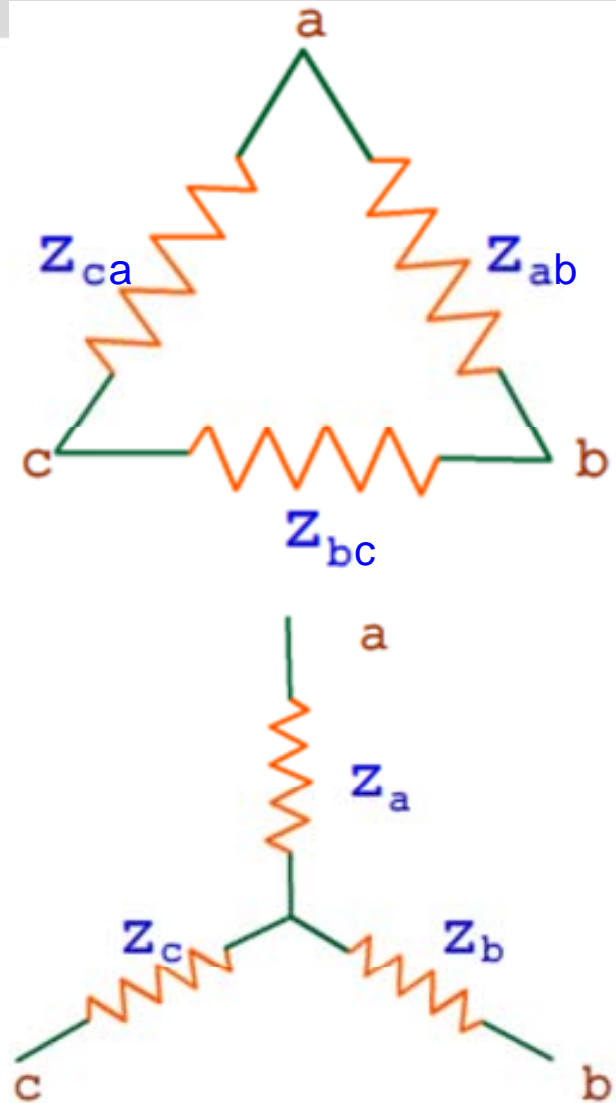
The reverse process converts  $\Delta$ -connected impedances to an equivalent Y

$\Delta$  to Y transformation

$$Z_a = \frac{Z_{ab} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$Z_b = \frac{Z_{ab} Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

$$Z_c = \frac{Z_{bc} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$$



# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations

When generators and motors with known emfs are connected through a network containing several nodes, the output of machines can be found by eliminating all the nodes in the network except the nodes to which the emfs are connected

In the resulting mesh, each emf is connected directly to every other emf through a single impedance

The current flowing through each of the impedances is the difference in potential between the two terminals of the impedance divided by the impedance

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# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations

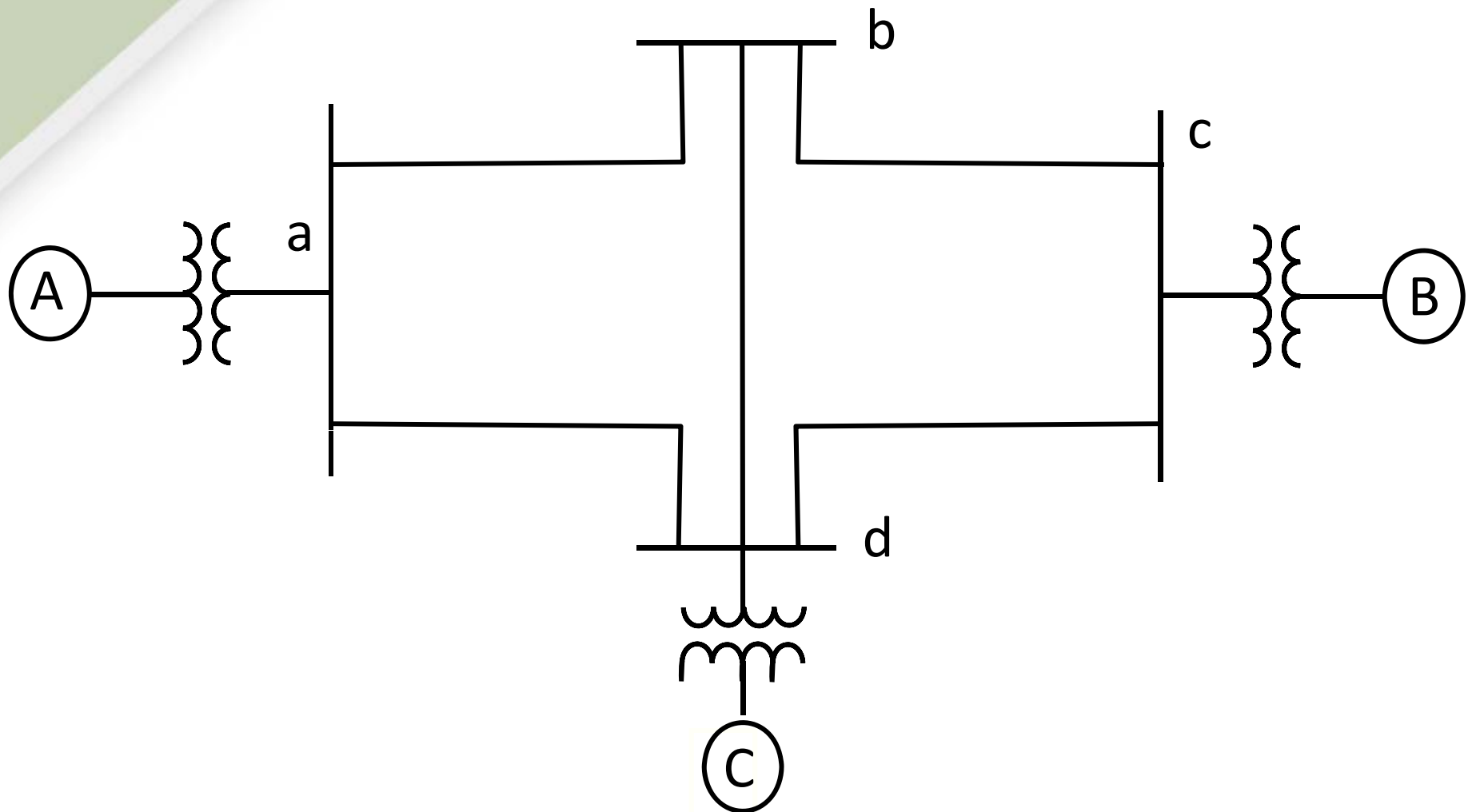
### Example:

Four high-tension buses labeled a, b, c, and d are connected as shown. Generators are connected to buses a and c and supply a synchronous motor load at bus d. The reactance diagram, with reactances specified in per unit, is shown in the figure. Simplify the circuit by eliminating all nodes except the neutral and e, f, and g, to which the emfs of the machines are connected. Note that the nodes preserved, other than the neutral, have no physical existence in the system

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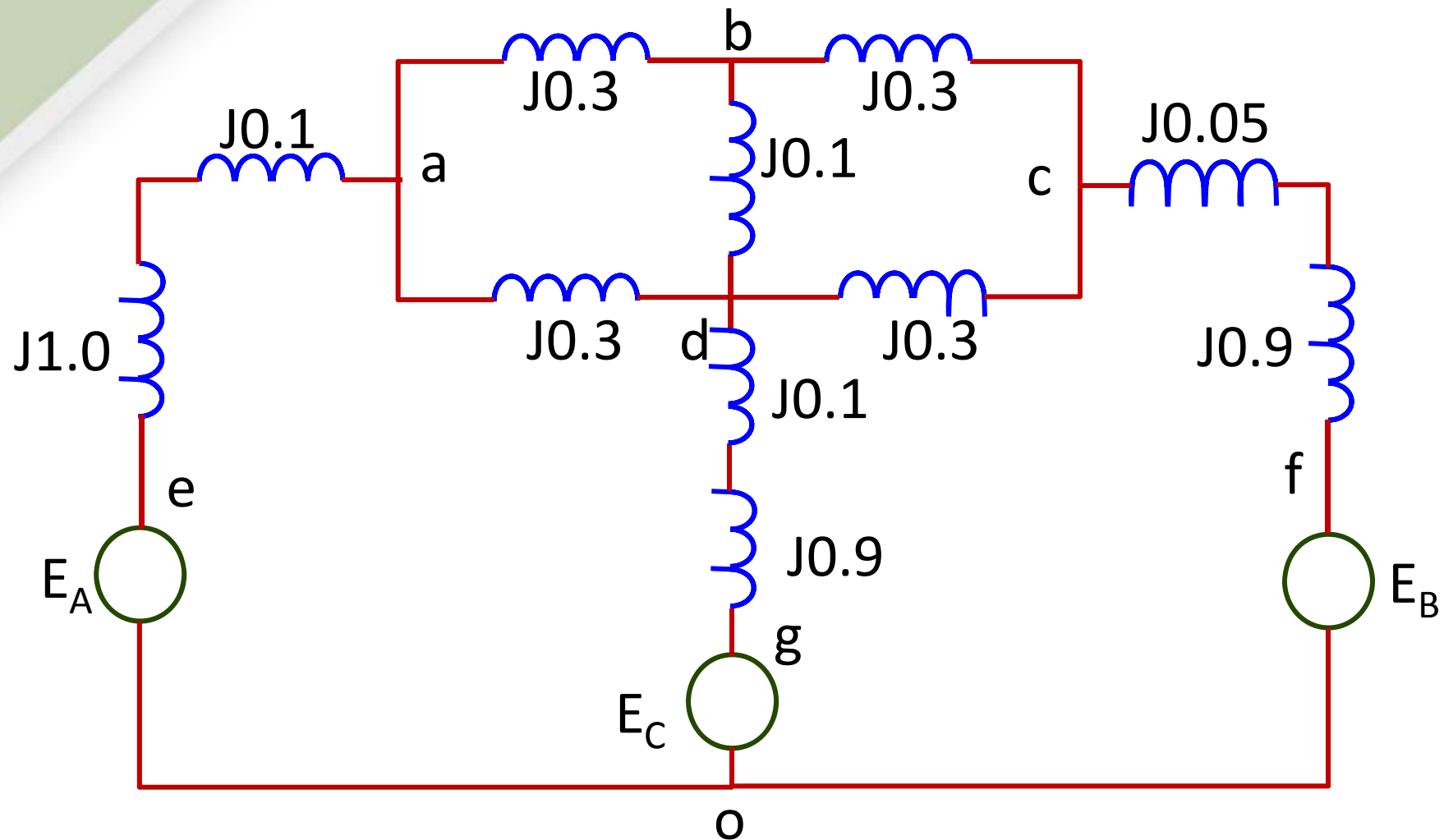
# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations



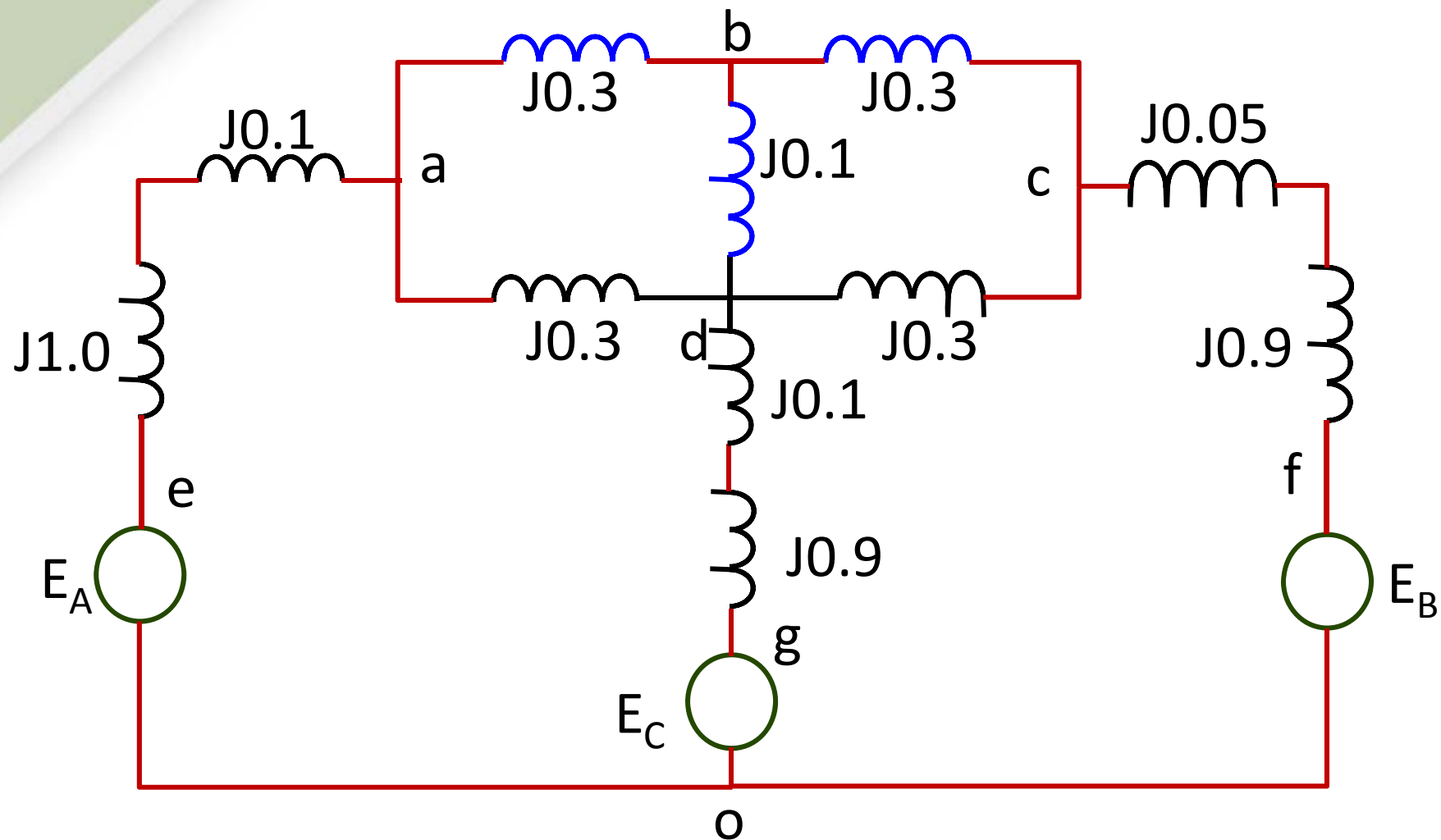
# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations



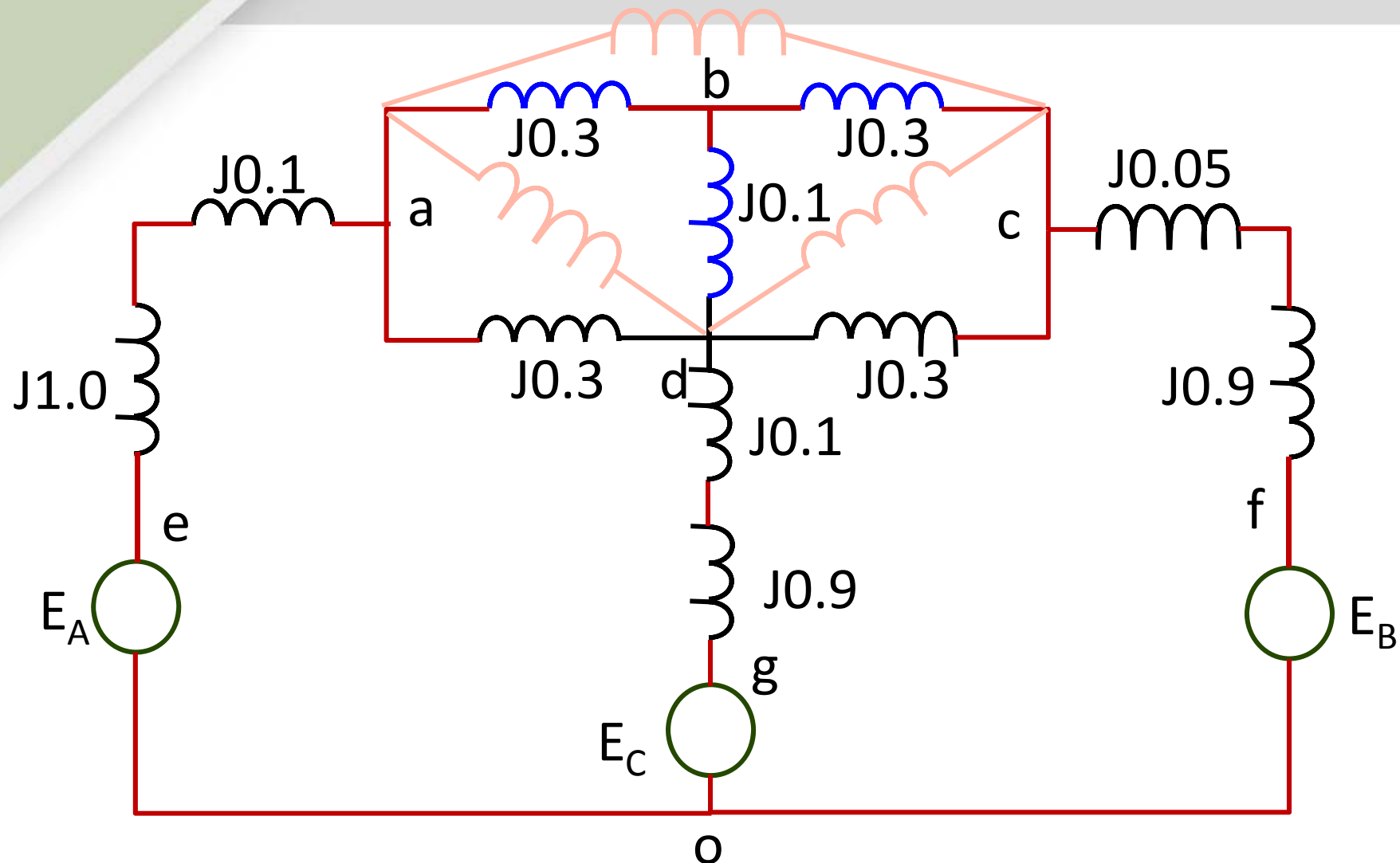
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## Node Elimination by Star-Mesh Transformations



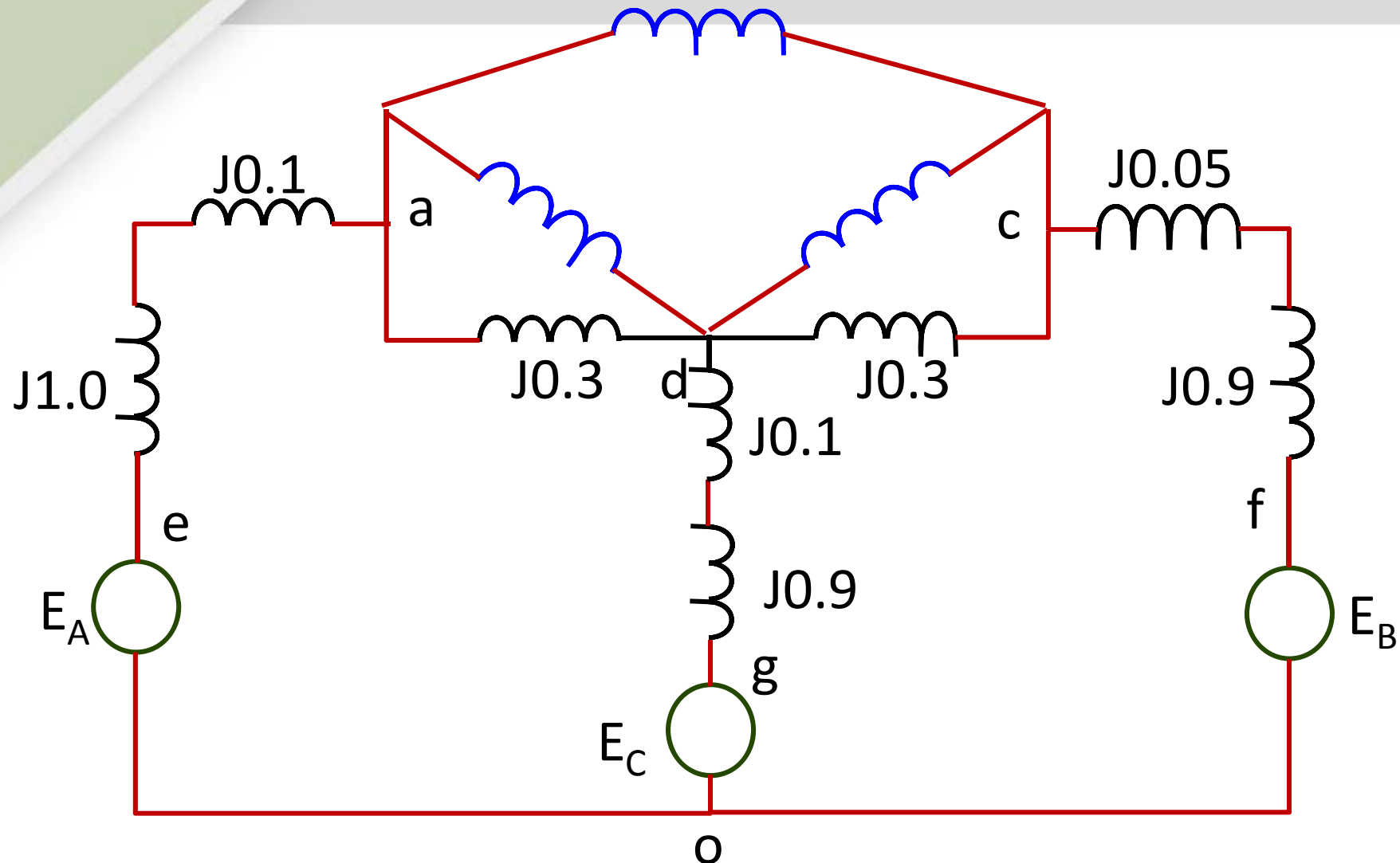
# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations



# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations



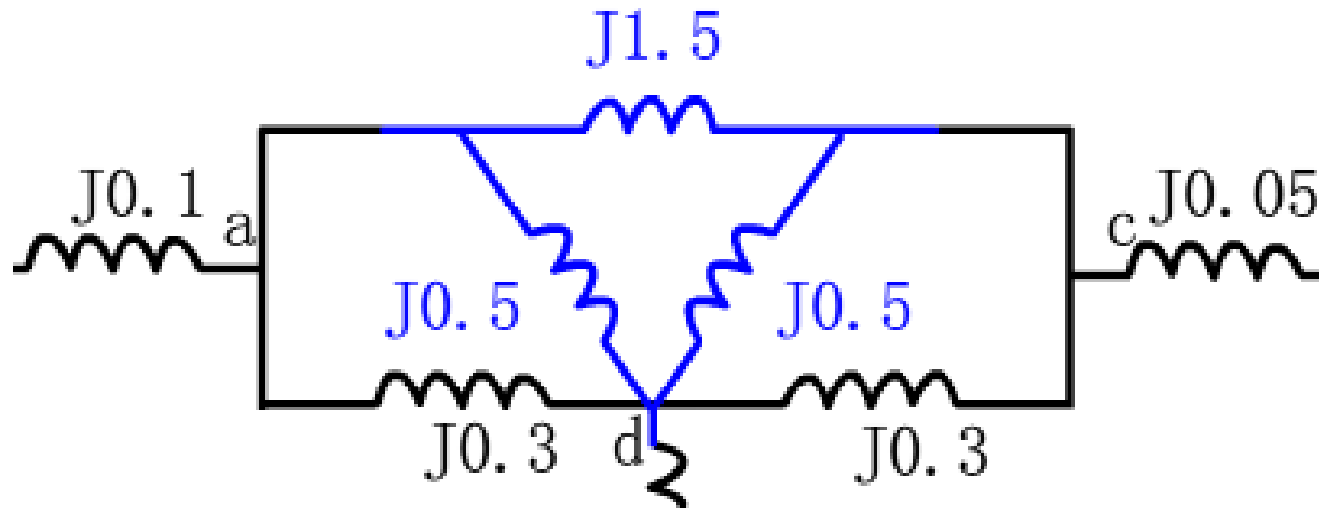
# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations

$$Z_{ac} = \frac{j0.3 \times j0.3 + j0.3 \times j0.1 + j0.3 \times j0.1}{j0.1} = \frac{-0.15}{j0.1} = j1.5 \text{ p.u.}$$

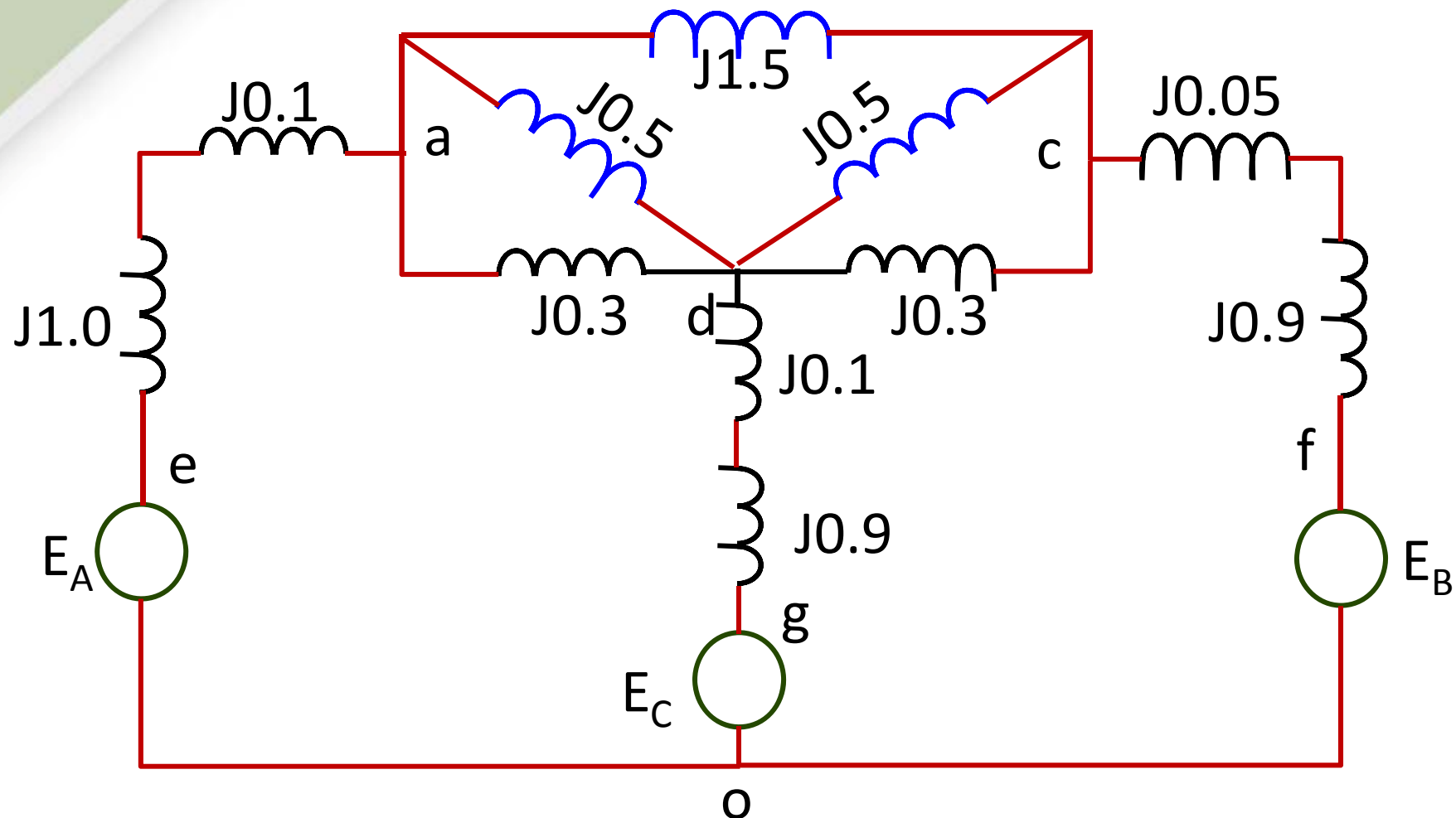
$$Z_{cd} = \frac{-0.15}{j0.3} = j0.5 \text{ p.u.}$$

$$Z_{da} = \frac{-0.15}{j0.3} = j0.5 \text{ p.u.}$$



# Network Equations and Solution

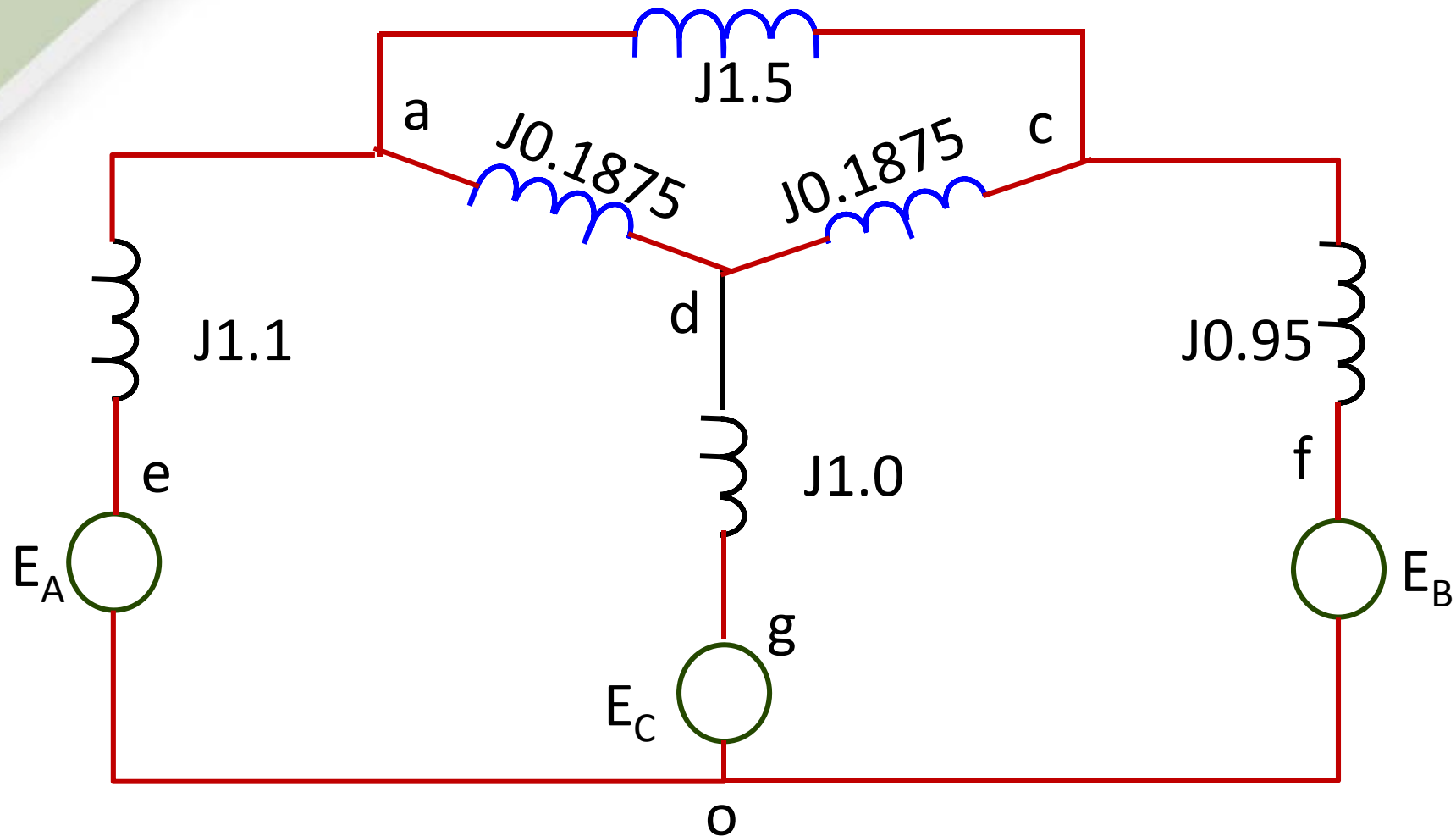
## Node Elimination by Star-Mesh Transformations





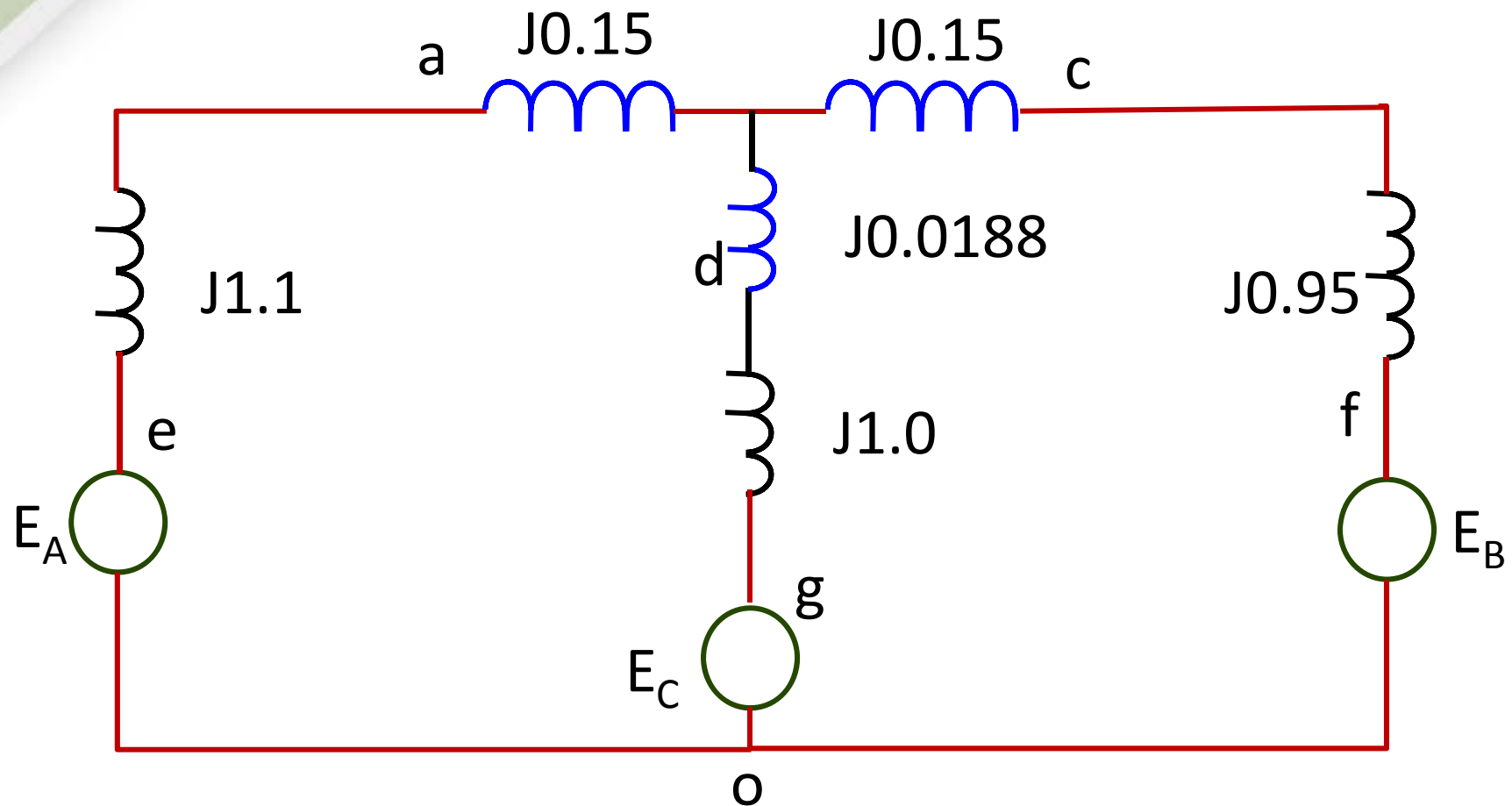
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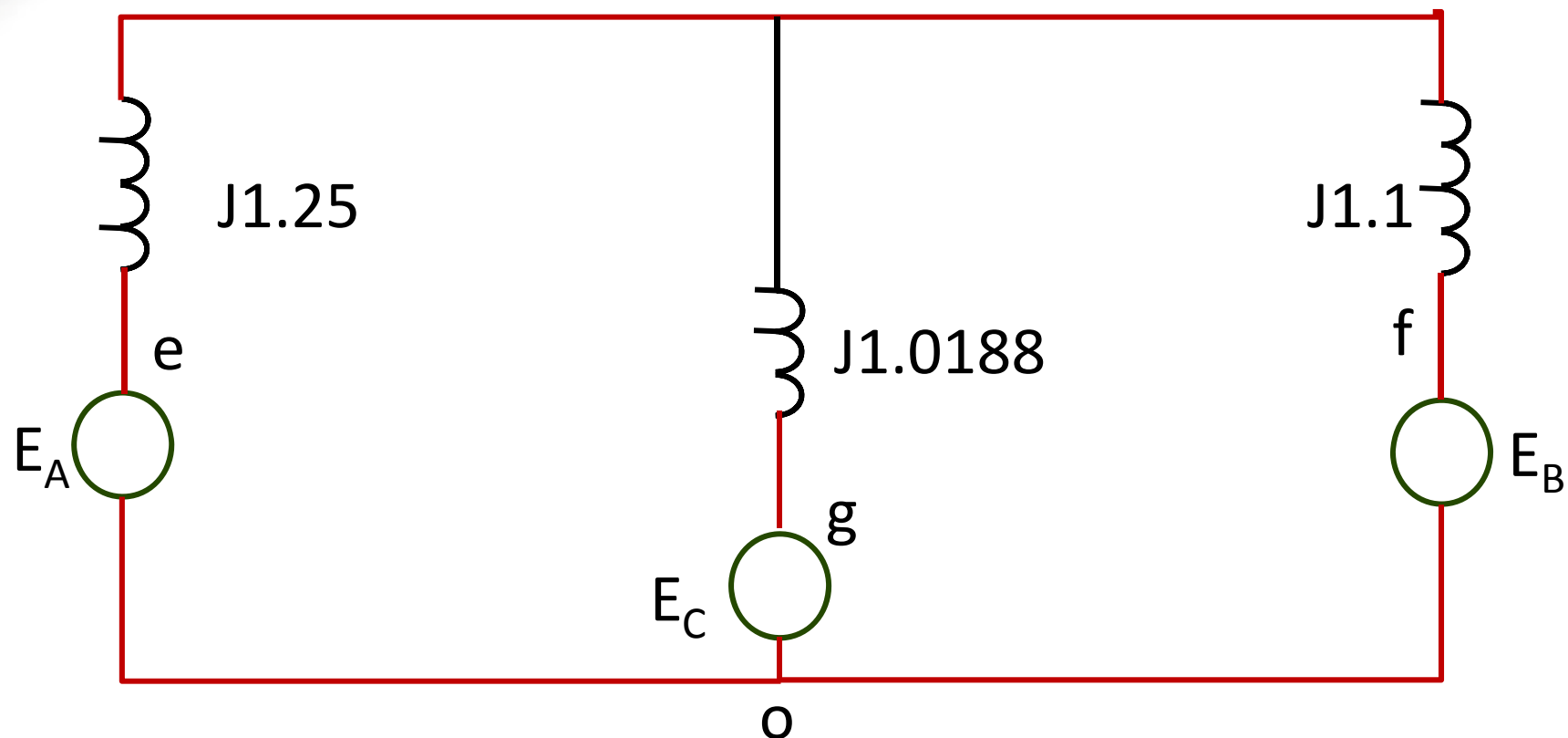
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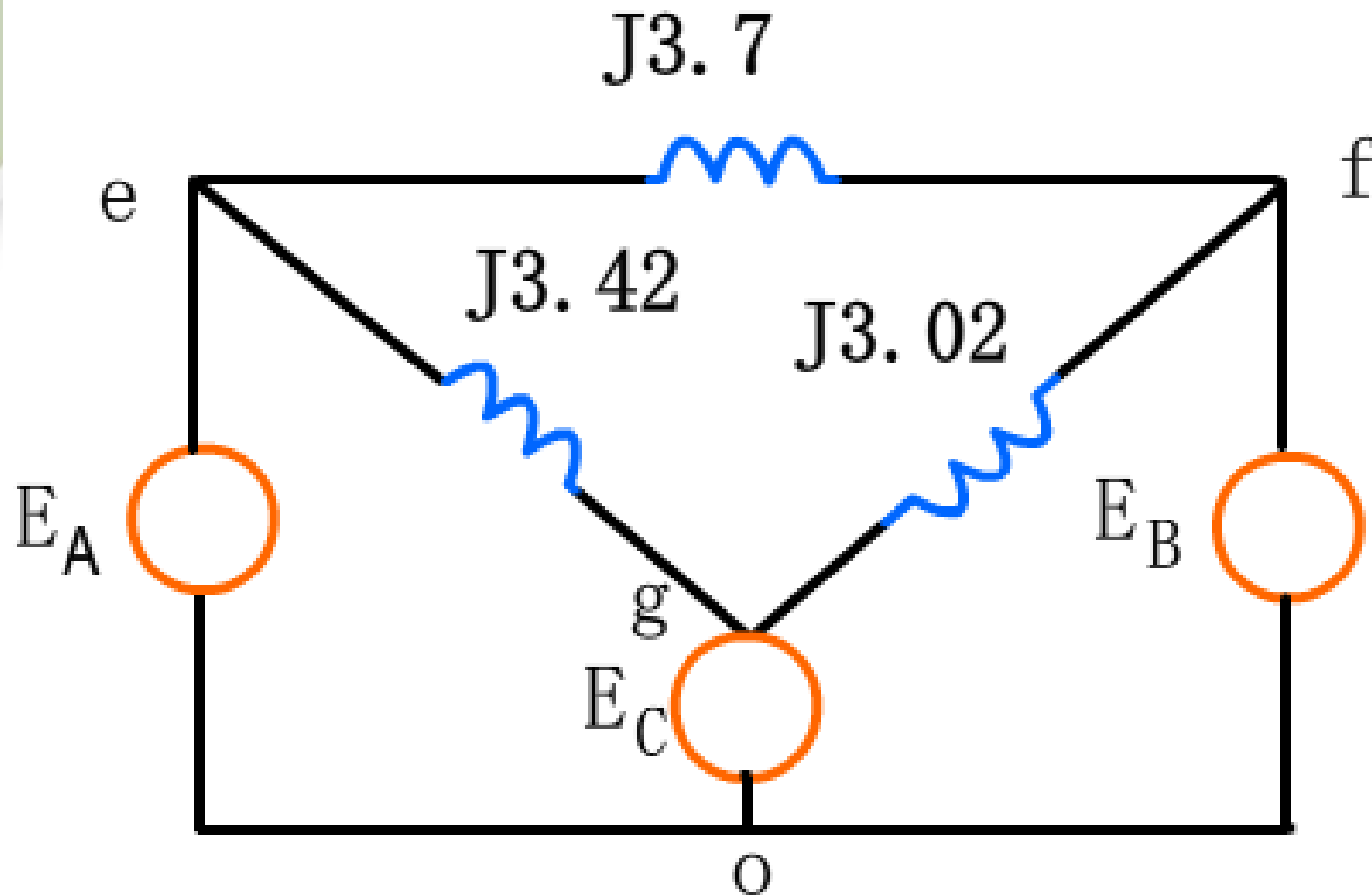
# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations



# Network Equations and Solution

## Node Elimination by Star-Mesh Transformations



# Network Equations and Solution

## Loop Equations

Each path between a pair of nodes in the circuit is called a **branch**

It is convenient to consider only those nodes to which more than two elements are connected and to call these junction points ***major nodes***

For a system having five nodes and eight branches, 16 equations are required to find the voltage across each branch and all currents for all branches

These equations could be solved simultaneously with complicated algorithms

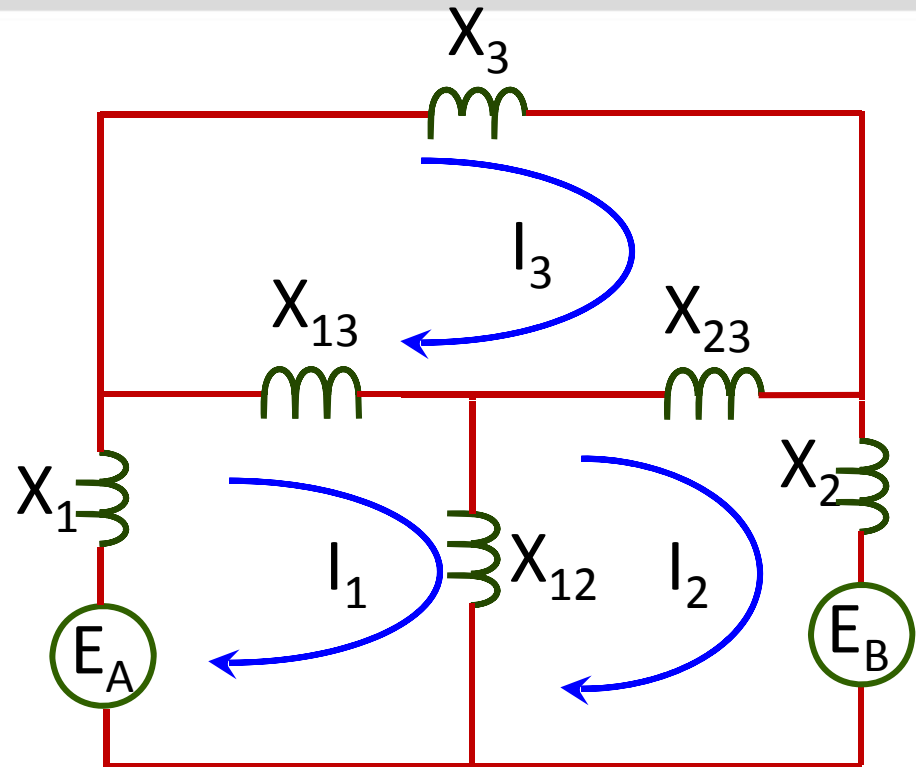
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# Network Equations and Solution

## Loop Equations

- Assume a current in each loop
- For each loop, apply Kirchhoff's voltage law

$$\sum \text{emf} = \sum IX$$



$$E_1 = Z_1 I_1 + Z_{12}(I_1 - I_2) + Z_{13}(I_1 - I_3) = (Z_1 + Z_{12} + Z_{13}) I_1 - Z_{12} I_2 - Z_{13} I_3$$

# Network Equations and Solution

## Loop Equations

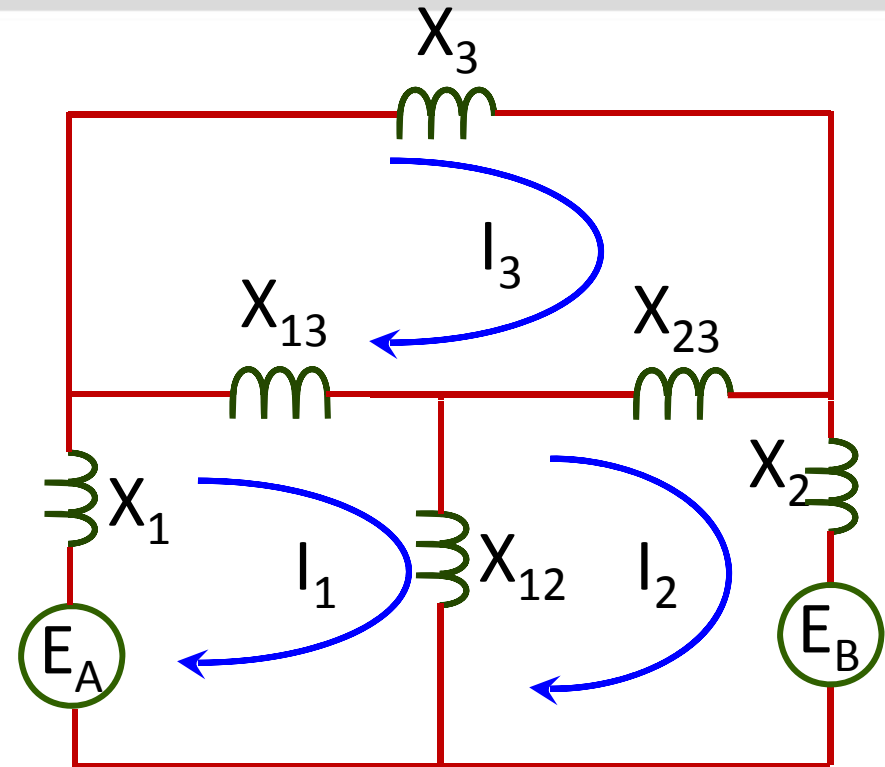
$$E_1 = Z_{11} I_1 - Z_{12} I_2 - Z_{13} I_3$$

$$E_2 = -Z_{21} I_1 + Z_{22} I_2 - Z_{23} I_3$$

$$E_3 = -Z_{31} I_1 - Z_{32} I_2 + Z_{33} I_3$$

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & -Z_{12} & -Z_{13} \\ -Z_{21} & Z_{22} & -Z_{23} \\ -Z_{31} & -Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\mathbf{E} = \mathbf{Z}_{\text{loop}} \mathbf{I}$$



$$\mathbf{I} = \mathbf{Z}_{\text{loop}}^{-1} \mathbf{E}$$